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# **Eddy Current and Field Gradient Degradation During a $\gamma_t$ Jump**

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# Eddy Current and Field Gradient Degradation During a $\gamma_t$ Jump

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## Abstract

An elliptical beam pipe is placed inside a quadrupole with rapidly changing field gradient. The eddy-current induced quadrupole field gradient inside the beam pipe is derived together with the AC eddy-current power loss at the walls of the beam pipe. The field gradient degradation inside the beam pipe is discussed and the expectations are compared with experimental measurements.

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## I. INTRODUCTION

During a  $\gamma_t$  jump designed for the Fermilab Main Injector, the special jump quadrupoles will be ramped in a time duration of  $\tau_0 \approx 0.5$  ms. Eddy current will be produced in the walls of the beam pipe leading to two consequences. First, the eddy current will produce an AC power loss. Second, the eddy current will generate a field gradient  $B'_{ed}$  which counteracts the quadrupole field gradient  $B'_{ext}$  produced by the external quadrupole current windings. This will lead to field gradient degradation and an elongation of the designed rise time of the  $\gamma_t$  jump. In this paper, we compute in Sec. II the AC power loss of the eddy current in the walls of an elliptical beam pipe as well as the quadrupole field induced inside. The degradation of the penetrating quadrupole field is discussed in Sec. III for two different types of time variations. Finally, in Sec. IV, the theoretical expectations are compared with an experimental measurement performed at the Brookhaven National Laboratory.

## II. EDDY CURRENT

We assume that the jump quadrupole has iron poles with magnetic permeability  $\mu = \infty$  so shaped that the field it produces is  $\vec{B} = B'_{ext}(y\hat{x} + x\hat{y})$ , or exactly quadrupole. The quadrupole is assumed to be infinite in the longitudinal direction so that all end effects can be neglected. The radial component of the magnetic field at any point, denoted by the polar coordinates  $(r, \theta)$ , along the longitudinal line  $A$  in Fig. 1 can be written as

$$B_r(\theta) = B'_{ext}r \sin 2\theta . \quad (2.1)$$

Note that Eq. (2.1) is symmetric with respect to the  $45^\circ$  radial line  $\theta = \pi/4$ . Therefore, the magnetic field at the longitudinal line  $B$ , where  $\theta \rightarrow \pi - \theta$ , has exactly the same radial component. Join  $A$  and  $B$  by the circular surface of a cylinder centered along the  $z$ -axis. Now, as the magnetic field gradient  $B'_{ext}$  is changing, the longitudinal electric field  $E_z$  at  $A$ , which is the same but opposite at  $B$ , can be obtained using Faraday's law by integrating the radial component of the magnetic field over the surface  $AB$ . Thus,

$$E_z(\theta) = \int_{\theta}^{\pi/4} \dot{B}'_{ext}r \sin 2\theta r d\theta = \frac{1}{2}\dot{B}'_{ext}r^2 \cos 2\theta , \quad (2.2)$$

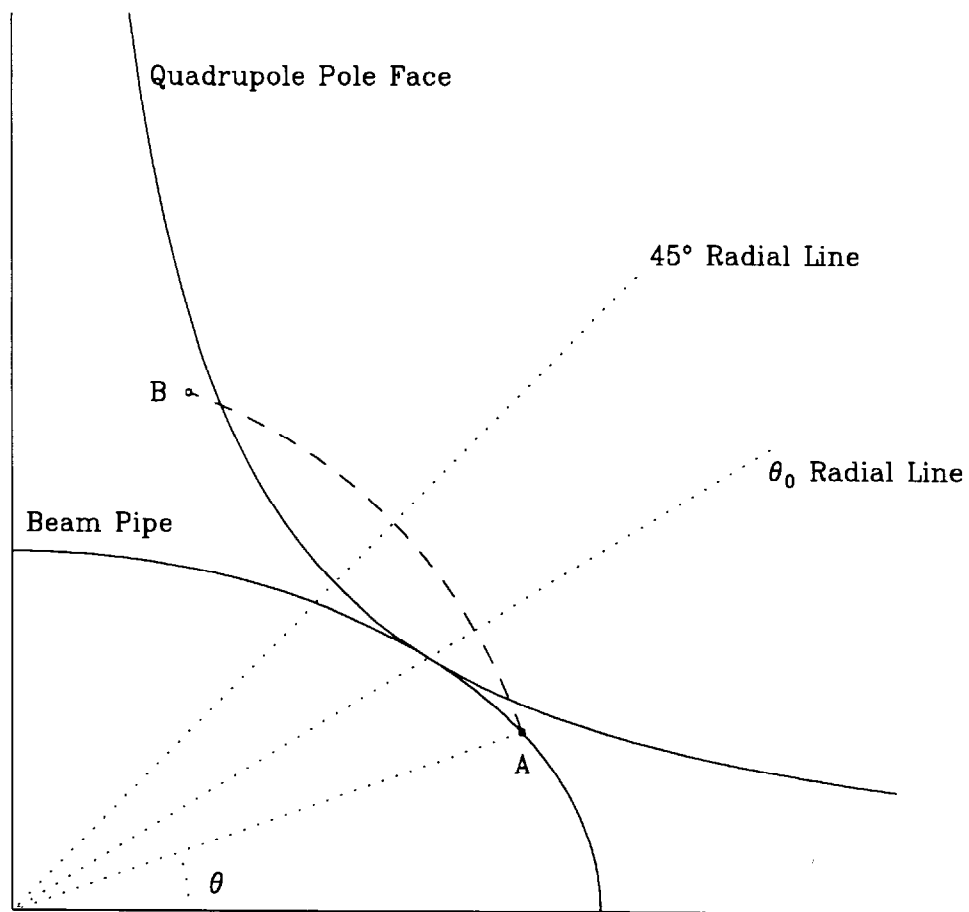


Figure 1: The geometry of the elliptical beam pipe and the pole face of the quadrupole.

where  $r$  is the radius of the cylindrical surface  $AB$ .

We now bring in the elliptical beam pipe which fits snugly touching the magnet poles. The beam pipe has horizontal and vertical radii  $a$  and  $b$ . Consider the longitudinal line  $A$  to be on the surface of the beam pipe. Then, the eddy current along a longitudinal strip at  $A$  of width  $d\ell$  of the beam pipe surface is

$$dI(\theta) = \sigma E_z(\theta) u d\ell , \quad (2.3)$$

where  $u$  is the wall thickness of the beam pipe and  $\sigma$  its conductivity. Choosing the parameterization of the elliptic cross section as

$$x = a \cos \varphi , \quad y = b \sin \varphi , \quad (2.4)$$

we can write

$$d\ell = \sqrt{dx^2 + dy^2} = \sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi} d\varphi . \quad (2.5)$$

The polar angle  $\theta$  and the ellipse parameterization angle  $\varphi$  can be easily related by

$$\tan \theta = \frac{y}{x} = \frac{b}{a} \tan \varphi . \quad (2.6)$$

The eddy current will in turn produce an induced magnetic field at the bunch inside the beam pipe. Here, we assume that all higher multipoles of the induced field are small enough so that they can be neglected. In other words, the induced field will be quadrupole denoted by  $\vec{B} = B'_{\text{ed}}(y\hat{x} + x\hat{y})$  and its radial component given by Eq. (2.2) with the subscript ‘ext’ replaced by ‘ed’. To determine  $B'_{\text{ed}}$ , we employ Ampere’s law by equating the integration of the induced magnetic field along a radial line at a angle  $\theta_0$ , where the beam pipe touches the magnet pole, and then return along the  $x$ -axis to the eddy current enclosed. First, the determination of the angle  $\theta_0$ . The cross section of the pole face is given by

$$xy = f \quad (2.7)$$

where  $f$  is a constant. The intersection of the pole and the beam pipe leads to

$$ab \sin \varphi_0 \cos \varphi_0 = f , \quad (2.8)$$

while the equality of the tangents at the touching point leads to

$$ab \frac{\cos^3 \varphi_0}{\sin \varphi_0} = f . \quad (2.9)$$

Therefore,

$$\varphi_0 = \frac{\pi}{4} \quad \text{and} \quad \theta_0 = \tan^{-1} \frac{b}{a}. \quad (2.10)$$

The integration of the induced field along the  $\theta_0$  line gives

$$\int_0^{r_0} B_r dr = \int_0^{r_0} B'_{\text{ed}} r \sin 2\theta_0 = \frac{1}{2} ab B'_{\text{ed}} \quad (2.11)$$

where

$$r_0 = \sqrt{\frac{a^2 + b^2}{2}}$$

is the radial width of the beam pipe at angle  $\theta_0$  and we have made use of the fact that  $\mu = \infty$  inside the iron pole. The integral along the  $x$ -axis is zero because the quadrupole field has no radial component there.

The contribution of the eddy current along the wall of the beam pipe can be obtained by integrating  $dI(\theta)$  in Eq. (2.3), or

$$\int_0^{\theta_0} dI(\theta) = \frac{1}{2} \sigma \dot{B}'_{\text{ext}} u \int_0^{\pi/4} (a^2 \cos^2 \varphi - b^2 \sin^2 \varphi) \sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi} d\varphi. \quad (2.12)$$

From Ampere's law, we finally obtain the induced field gradient,

$$B'_{\text{ed}} = -\frac{1}{2} \mu_0 \sigma \dot{B}'_{\text{ext}} u b F_{\text{ed}}\left(\frac{a}{b}\right), \quad (2.13)$$

where  $\mu_0$  is the magnetic permeability in vacuum and  $F_{\text{ed}}(\frac{a}{b})$  is the form factor which takes the value of unity when  $a = b$  for a circular pipe. The form factor can be written as

$$F_{\text{ed}}(x) = \frac{2}{x} \int_0^{\pi/4} (x^2 \cos^2 \varphi - \sin^2 \varphi) \sqrt{x^2 \sin^2 \varphi + \cos^2 \varphi} d\varphi. \quad (2.14)$$

which can be integrated to elliptical functions, and is plotting in Fig. 2.

The AC power dissipated by the eddy current can be computed easily. In an element  $d\ell$  of the wall of the beam pipe, the current is  $J_z u d\ell$  where the density of the eddy current is  $J_z = \sigma E_z$ . The AC power dissipated per unit longitudinal length in the beam pipe is therefore

$$P = \int \frac{(J_z u d\ell)^2}{\sigma u d\ell}. \quad (2.15)$$

Substituting Eqs. (2.2), (2.3), and (2.5), we obtain

$$P = \frac{1}{4} \pi \sigma \dot{B}'_{\text{ext}}{}^2 u b^5 F_{\text{p}}\left(\frac{a}{b}\right), \quad (2.16)$$

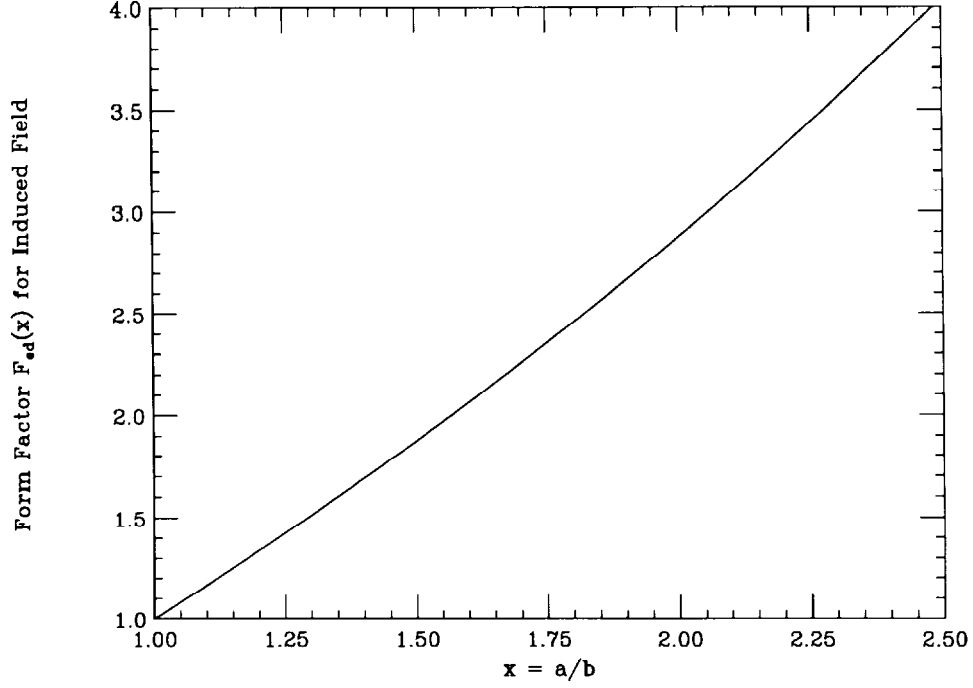


Figure 2: The form factor of the field gradient induced by the eddy current on the walls of an elliptical beam pipe.

where the form factor

$$F_F(x) = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} (x^2 \cos^2 \varphi - \sin^2 \varphi)^2 \sqrt{x^2 \sin^2 \varphi + \cos^2 \varphi} d\varphi, \quad (2.17)$$

is set to unity when  $x = 1$  for a circular pipe, and is plotted in Fig. 3.

In the consideration of the  $\gamma_t$ -jump design of the Main Injector, the elliptical beam pipe has  $a/b = 1.56$ , which will be elongated to  $a/b \approx 2.00$  under vacuum. This gives  $F_{ed} = 1.99$  to  $2.88$  and  $F_p = 3.13$  to  $10.34$ . This does not mean that the eddy-current AC power loss will increase by three times under vacuum, because the height of the beam pipe  $2b$  will be shortened under vacuum and the AC power loss is proportional to  $b^5$  as indicated in Eq. (2.15).

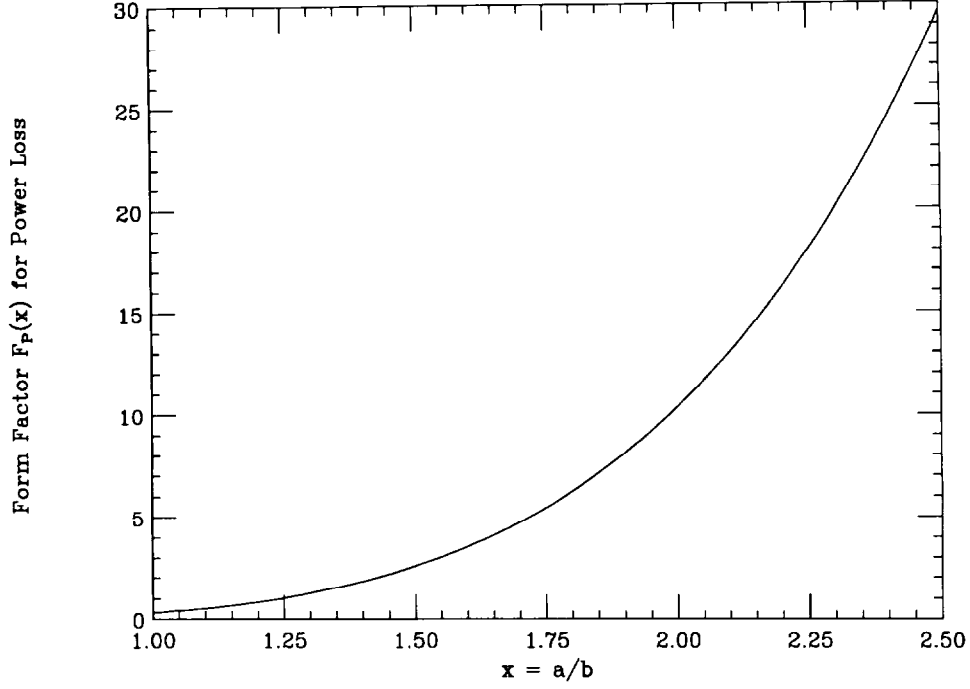


Figure 3: The form factor of the AC power dissipated by the eddy current in the walls of an elliptical beam pipe.

### III. FIELD GRADIENT DEGRADATION

The field gradient degradation inside the beam pipe depends very critically on how the external field is changing. First, let us consider a sinusoidal variation of the external field gradient:

$$B'_{\text{ext}}(t) = B'_0 e^{-i\omega t}, \quad (3.1)$$

where  $t$  denotes time. Since the eddy-current-induced quadrupole field gradient  $B'_{\text{ed}}$  of Eq. (2.13) is proportional to  $\dot{B}'_{\text{ext}}$ , it is therefore  $90^\circ$  out of phase with  $B'_{\text{ext}}$ . A more careful derivation by matching boundary conditions at the beam pipe leads to [1]

$$\frac{B'_{\text{in}}}{B'_{\text{ext}}} = \frac{B'_{\text{ext}} + B'_{\text{ed}}}{B'_{\text{ext}}} = \frac{1}{1 - i\omega\tau} = \cos(\omega\tau) e^{i\omega\tau}, \quad (3.2)$$

where  $B'_{\text{in}}$  is the field gradient inside the beam pipe and

$$\tau = \frac{1}{2} ub\mu_0\sigma F_{\text{ed}}\left(\frac{a}{b}\right) \quad (3.3)$$

is the characteristic time delay of the beam pipe. One can easily verify that Eq. (3.2) is the same as Eq. (2.13) provided that

$$|B'_{\text{ed}}| \ll |B'_{\text{ext}}|. \quad (3.4)$$

By the way, we have always been working in a perturbative approach, which is valid only if the condition in Eq. (3.4) is satisfied. Therefore, the response to the external field gradient is

$$B'_{\text{in}} = B'_0 \cos(\omega\tau) e^{-i\omega(t-\tau)}. \quad (3.5)$$

Thus  $\tau$  actually represents a delay. The field gradient degradation is therefore

$$\left| \frac{B'_{\text{ed}}}{B'_0} \right| \approx \frac{1}{2}(\omega\tau)^2. \quad (3.6)$$

If we consider a rise time of  $\tau_0 = 0.5$  ms, it is reasonable to let  $\omega \approx \tau_0^{-1}$ . For a circular stainless steel pipe of radius  $b = 3$  in and wall thickness  $u = 0.0125$  in, this degradation amounts to

$$\left| \frac{B'_{\text{ed}}}{B'_0} \right| \approx \frac{1}{2} \left( \frac{\tau}{\tau_0} \right)^2 = \frac{1}{2} \left( \frac{ub\mu_0\sigma}{\tau_0} \right)^2 = \frac{1}{2} \left( \frac{ub}{\delta^2} \right)^2 = 0.34\%, \quad (3.7)$$

which is extremely small. This is because the eddy-current-induced field gradient is  $90^\circ$  out of phase, and is therefore quadratic in  $\tau/\tau_0$ . In above,  $\delta$  is the skin depth of the walls of the beam pipe.

On the other hand, if the external field gradient behaves as

$$B'_{\text{ext}}(t) = \begin{cases} B'_0 & t < 0, \\ B'_0 e^{-t/\tau_0} & t \geq 0, \end{cases} \quad (3.8)$$

the eddy-current-induced field gradient  $B'_{\text{ed}}$  will be in phase (modulus  $\pi$ ) with  $B'_{\text{ext}}$ . We therefore have a field gradient degradation

$$\left| \frac{B'_{\text{ed}}}{B'_0} \right| = \frac{\tau}{\tau_0}, \quad (3.9)$$

which amounts to, for the above example of the stainless steel beam pipe, 8.2%, which is very much larger.

#### IV. MEASUREMENT

An experiment was performed at the Brookhaven National Laboratory using circular beam pipes of radius  $b = 3$  in and of various material and thickness placed inside a  $\gamma_t$ -jump quadrupole [2]. Initially, the quadrupole carries a constant field gradient  $B'_0$ . At time  $t = 0$ , the field was discharged exponentially through a resistor with a time constant  $\tau_0 = 0.5$  ms. The time variation of the external field gradient is therefore described by Eq. (3.8). A pick-up coil was placed inside the beam pipe to measure the rate at which the field gradient inside was changing, or  $\dot{B}'_{\text{in}}(t)$ , and was recorded as a voltage response  $V_r$  relative to  $V_0$ , the situation when there was no beam pipe.

Using Laplace transform and with the aid of Eq. (3.2), we can readily find the field gradient inside the beam pipe,

$$B'_{\text{in}}(t) = \begin{cases} B'_0 & t < 0, \\ -\frac{B'_0}{\tau_0 - \tau} (e^{-t/\tau_0} - e^{-t/\tau}) & t \geq 0, \end{cases} \quad (4.1)$$

The pick-up coil picked up  $\dot{B}'_{\text{in}}(t)$ , which is for  $t \geq 0$ ,

$$\dot{B}'_{\text{in}}(t) = -\frac{B'_0}{\tau_0 - \tau} (e^{-t/\tau_0} - e^{-t/\tau}). \quad (4.2)$$

Note that  $|\dot{B}'_{\text{in}}(u)|$  rises from zero at  $t = 0$ , attains a maximum, and falls to zero again gradually. It is easy to show that the maximum occurs at

$$t = \frac{\tau\tau_0}{\tau_0 - \tau} \ln \frac{\tau_0}{\tau}, \quad (4.3)$$

and assumes the value

$$|\dot{B}'_{\text{in}}|_{\text{max}} = \frac{B'_0}{\tau_0} \left( \frac{\tau}{\tau_0} \right)^{\tau/(\tau_0 - \tau)}. \quad (4.4)$$

Therefore, the effect of the beam pipe is a decrease and a delay in the measured peak voltage. In the limit of  $\tau \rightarrow 0$ , or when there is no beam pipe, Eq. (4.4) gives

$$|\dot{B}'_{\text{in}}|_{\text{max}} \rightarrow \frac{B'_0}{\tau_0}. \quad (4.5)$$

Thus, the relative response monitored by the pick-up coil is

$$\frac{V_r}{V_0} = \left( \frac{\tau}{\tau_0} \right)^{\tau/(\tau_0 - \tau)}. \quad (4.6)$$

Some of the measurements are shown in Fig. 4. The top oscilloscope photograph is the situation without any beam pipe. The lower trace represents the quadrupole current discharging through a resistor, while the top trace represents the rate of change of magnetic field gradient inside the beam pipe,  $|\dot{B}'_{\text{in}}|$ , which registered as  $V_0 = 6.210$  V through the pick-up coil. The time scale is  $50\mu\text{s}$  per division and there are 10 divisions in total. Although we expect no delay for the response theoretically, we notice that there is a delay of roughly  $75\mu\text{s}$ , which might have been due to the switching system. The lower photograph is the situation when an aluminum beam pipe of wall thickness 0.125 in was present. Here, the time scale is  $500\mu\text{s}$  per division. We clearly see that the sharp exponential decay of the discharging quadrupole current and the rise of the response voltage  $V_r$ , which is proportional to  $|\dot{B}'_{\text{in}}|$  inside the beam pipe. Notice that  $V_r$  shows a maximum of 0.5459 V with a delay of roughly 1 ms.

The experimental results and theoretical predictions are tabulated in Table I. For the theoretical expectations listed in the table, the resistivities  $\rho = 1.29, 0.74$ , and  $0.0265 \mu\Omega\text{-m}$  have been used for Inconel, stainless steel, and aluminum, respectively. We see that the maximum values of the response voltages are very well predicted, but the delays differ by a factor of 2. We would like to point out that theoretical predictions for the aluminum beam pipe are not believable. This is because  $\tau/\tau_0 > 1$ , which falls outside the validity of our perturbative method.

Table I: Induced field gradient inside a beam pipe subjected to a sudden shut off of an external quadrupole field.

Material	Thickness $u$	$\tau/\tau_0$	Time delay		$V_r/V_0$	
			Theory	Expt.	Theory	Expt.
Inconel	0.020 in	0.038	$64 \mu\text{s}$	$0 \mu\text{s}$	0.88	0.90
Stainless Steel	0.025 in	0.082	$110 \mu\text{s}$	$50 \mu\text{s}$	0.80	0.76
Stainless Steel	0.062 in	0.20	$200 \mu\text{s}$	$100 \mu\text{s}$	0.67	0.68
Aluminum	0.125 in	11.5	$1140 \mu\text{s}$	$1000 \mu\text{s}$	0.07	0.09

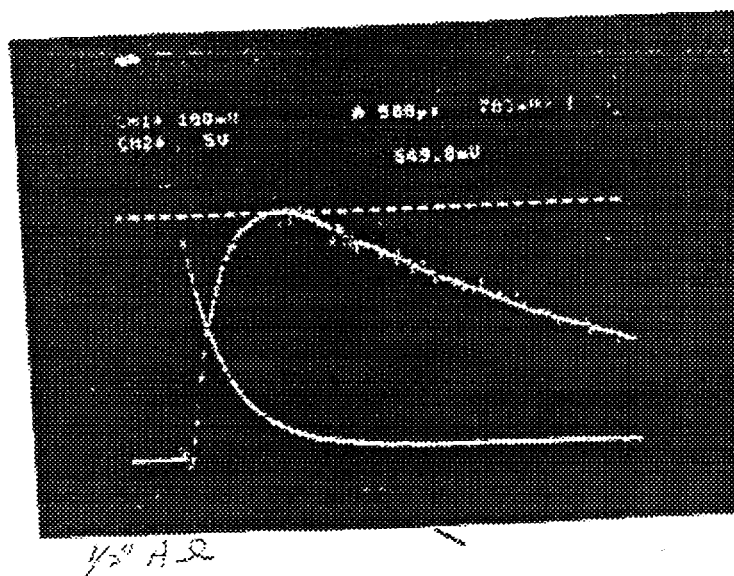
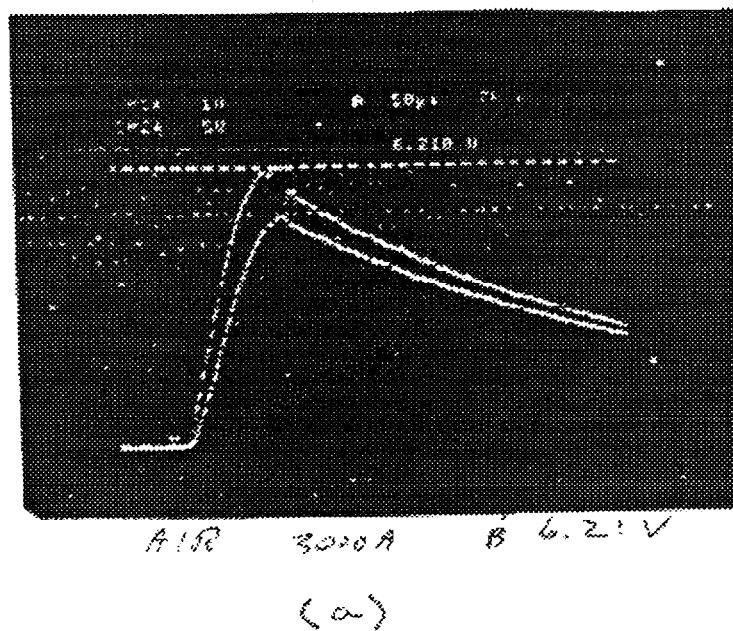


Figure 4. Experimental results showing the quadrupole discharging current and the response voltage, which is proportional to  $|\dot{B}'_{in}|$  inside the beam pipe. The top photograph is the situation without a beam pipe and the lower one is the situation when an aluminum beam pipe of wall thickness 0.125 in was present.

## References

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- [2] A. Feltman, J. Funaro, L. Ratner, W. van Asselt, P. Yamin, *Eddy Currents in the Transition Jump Quadrupole Vacuum Chambers*, BNL Internal Report, AGS/AD/Tech. Note No.. 332, 1989.